California State University
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ALGEBRA — EXPONENTS

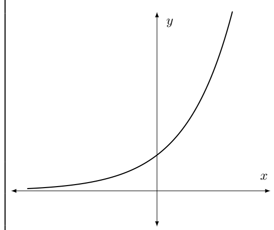
DEFINITION

An **exponential function** is a function of the form

$$f(x) = a \cdot b^x$$

where a and b are constants and x is the input variable.

The function $f(x) = b^x$ with $b > 0$ has a **domain** of $(-\infty, \infty)$ and a **range** of $(0, \infty)$.



RELATIONSHIP

A **logarithm** is the inverse operation to **exponentiation**. Therefore

$$\mathbf{a^x = b} \qquad \text{can be written as} \qquad \mathbf{\log_a(b) = x}$$

Therefore both statements ask for a power x such that a raised to that power equals b.

PROPERTIES OF EXPONENTS

RULE	EXAMPLE
$a^0 = 1$	$4^0 = 1$
$a^1 = a$	$5^1 = 5$
$\frac{1}{a^x} = a^{-x}$	$\frac{1}{2^5} = 2^{-5}$
$a^x \cdot a^y = a^{x+y}$	$2^3 \cdot 2^4 = 2^{3+4}$
$\frac{a^x}{a^y} = a^{x-y}$	$\frac{3^5}{3^4} = 3^{5-4}$
$(a^x)^y = a^{x \cdot y}$	$(4^3)^5 = 4^{15}$
$a^{\log_a(x)} = x$	$14^{\log_{14}(5)} = 5$

ALGEBRA LOGARITHMS

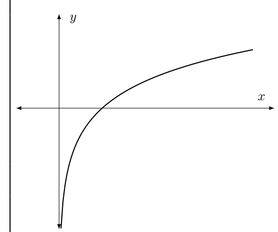
DEFINITION

An **logarithmic function** is a function of the form

$$f(x) = a \cdot \log_b(x)$$

where a and b are constants and x is the input variable.

The function $f(x) = \log_b(x)$ with $b > 0$ has a **domain** of $(0, \infty)$ and a **range** of $(-\infty, \infty)$.



NATURAL LOGARITHM

The **natural logarithm** is a logarithm with base e

$$\log_e(x) = \ln(x)$$

CHANGE OF BASE FORMULA

$$\log_a(b) = \frac{\log_{10}(b)}{\log_{10}(a)} = \frac{\ln(b)}{\ln(a)}$$

PROPERTIES OF LOGARITHMS

The following rules apply to the natural logarithm $\mathbf{ln(x)}$.

RULE	EXAMPLE
$\log_b(1) = 0$	$\log_{10}(1) = 0$
$\log_b(b) = 1$	$\log_{10}(10) = 1$
$\log_b\left(\frac{1}{u}\right) = -\log_b(u)$	$\log_7\left(\frac{1}{8}\right) = -\log_7(8)$
$\log_b(u \cdot v) = \log_b(u) + \log_b(v)$	$\log_5(7 \cdot 8) = \log_5(7) + \log_5(8)$
$\log_b\left(\frac{u}{v}\right) = \log_b(u) - \log_b(v)$	$\log_7\left(\frac{8}{9}\right) = \log_7(8) - \log_7(9)$
$\log_b(a^c) = c \cdot \log_b(a)$	$\log_5(6^7) = 7 \cdot \log_5(6)$
$\log_b(\sqrt[c]{a}) = \frac{1}{c} \cdot \log_b(a)$	$\log_8(\sqrt[3]{9}) = \frac{1}{3} \cdot \log_8(9)$